SUMMARY OF THE RESEARCH PROPOSAL "GEOMETRY, TOPOLOGY AND COMPLEXITY OF HYPERGEOMETRIC FUNCTIONS IN SEVERAL COMPLEX VARIABLES" BY T.M. SADYKOV

The focus of the proposed research project is on mathematical structures that appear in the study of hypergeometric functions in several complex variables. We will mainly pay attention to the A-hypergeometric functions (HGF) introduced by Gelfand-Kapranov-Zelevinski and to the Horn HGF. These functions are multivariate versions of special functions in one variable like Gauss HGF, Legendre, Bessel functions etc. Polynomial instances of HGF are ubiquitous in approximation theory and mathematical physics, Chebyshev's polynomials being one of the widest known family of orthogonal polynomials of this kind.

As an important application of the general hypergeometric framework, we plan to study the number of zeros of Abelian integrals. The estimation of this number will contribute to the understanding of the crucial difficulties that lie within Hilbert's XVI-th problem on the number of limit cycles that appear in a perturbed planar Hamiltonian system. The proposed research will spin around the following topics:

1. Number and location of zeros of HGF's in one and several variables. With any integer convex polytope $P \subset \mathbb{R}^n$ we will associate a multivariate hypergeometric polynomial whose set of exponents is $\mathbb{Z}^n \cap P$. This polynomial will be defined uniquely up to a constant multiple and can be viewed as a solution to a holonomic system of partial differential equations. Under certain nondegeneracy conditions, the zero locus of any such polynomial will be shown to be optimal in the sense that its amoeba has the most complicated topology allowed by the structure of the Newton polytope.

2. The weighted moment map and the amoeba-like simplicial complex associated with an algebraic variety. We will introduce a simplicial complex that encodes intrinsic combinatorial properties of an algebraic hypersurface while possessing key properties of its compactified amoeba. This simplicial complex will be defined to be a subset of the Newton polytope of the defining polynomial and will allow to investigate its analytic and topological properties in combinatorial terms. This direction of research leads far beyond the hypergeometric world.

3. Topology of the complement to the singular divisors of solutions to holonomic systems of hypergeometric PDEs. For a deformation of an affine toric variety, the period integrals satisfy a Horn type hypergeometric system that can be described by means of the Newton polytopes of the defining equations of the variety. We plan to investigate this system and the topology of its singularities.

4. Analytic complexity of HGF. All known examples of bivariate hypergeometric functions are of finite analytic complexity. Yet, the holonomic systems of linear partial differential equations defining hypergeometric functions are very different from the differential polynomials that yield the analytic complexity classes. We plan to relate the analytic complexity of a bivariate HGF and the holonomic rank of its defining system of PDEs.

The expected results of the research include the following statements.

Theorem 1. *The zero locus of any strongly irreducible hypergeometric polynomial in $n \geq 2$ variables is an optimal algebraic hypersurface.*

Theorem 2. *The set-theoretical limit (as $k \to \infty$) of image of the k-th Hadamard power of a Laurent polynomial under the weighted moment is a simplicial complex that is isotopic to its compactified amoeba.*